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Robust Design Optimization of 3D Truss Structures using Evolutionary Computation

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Abstract: In engineering problems, the randomness and uncertainties are inherent and the scatter of structural parameters from their nominal ideal values is unavoidable. Robust Design Optimization (RDO) methods primarily seek to minimize the influence of stochastic variations on the mean design, and traditionally rely on rough approximations of the stochastic response about the mean design. RDO yields a design with a state of robustness, so that its performance is the least sensitive to the variability of uncertain variables. The Monte Carlo Simulation method, that has been employed in the present work, has been proven to be very efficient for studying the stochastic response of large-scale structural systems with a large number of random variables. In this study, the task of robust design optimization of structures is formulated as a multi-criteria optimization problem, in which the design variables of the optimization problem, together with other design parameters, such as the modulus of elasticity and the yield stress, are considered as random variables with a mean value equal to their nominal value. The weight of the structure as well as the variance of the structural response are to be minimized. The optimization algorithm employed is a two stage multi-membered Evolution Strategies scheme specially tailored for solving multi-criteria structural optimization problems.

Key words: Robust design, structural optimization, Evolution Strategies, Monte Carlo simulation.

INTRODUCTION

A typical engineering task during the development of any structural system is to improve its performance. Improvements can be achieved either by simply using design rules based on the experience or in a more automatic way by using optimization methods that lead to a structural design which is considered the optimum one. Strictly speaking optimal means that no better solution exists. Considering the complexity of the optimization problem to be solved it is obvious that finding the absolute optimum solution is a very difficult task. In the real world of structures, given the uncertainty or scatter of the structural parameters, the importance of such a computationally optimum solution would be limited. Although in a computing environment nearly perfect structural models can be simulated, real world structures always have imperfections or deviations from the nominal state. So the optimum that is obtained computationally will not be able to be exactly materialized, and as a result a near optimal solution is always implemented in practice. A deterministic based formulation of a structural optimization problem ignores scatter of any kind of parameters in order to build the so-called response surface. It is possible to find an optimum on that surface, but once this solution is transferred back to the physical system its optimality may vanish because of the parameters scatter which is unavoidable. Consequently, the performance of the 'real' design may be far worse than the expected one. In order to account for the randomness of some parameters that affect the response of the structure, a different formulation of the optimization problem has to be used. This formulation has to be based on stochastic analysis in order to take the random nature of some parameters of the structure into account.

In recent years, probabilistic based formulations of the optimization problem have been developed to account for uncertainty and randomness through stochastic simulation and probabilistic analysis. Stochastic analysis methods have been developed over the last two decades [1, 2] and have stimulated the interest for the probabilistic optimum design of structures. There are two distinguished design formulations that account for probabilistic systems response: Robust Design Optimization (RDO) [3-5] and Reliability-Based Design Optimization (RBDO) [6-8]. RDO methods primarily seek to minimize the influence of stochastic variations on the mean design, and traditionally rely on rough approximations of the stochastic response about the mean design, such as the First Order Second Moment methods. On the other hand, the main goal of RBDO methods is to design for safety with respect to extreme events and generally require a stochastic analysis of the system response far off the mean design such as Monte Carlo simulation or reliability methods. Despite the improvements achieved on the efficiency of the computational methods for treating reliability analysis problems, they still require disproportional computational effort for practical reliability problems. This is the reason why very few successful numerical investigations are known in the field [6].

In the present study the robust design sizing optimization of large-scale space trusses is investigated. The objective functions considered are the weight and the variance of the response of the structure, subject to stress and displacement constraints imposed by the design codes [9, 10]. Randomness of loads, material properties, and member geometry are taken into consideration in the stochastic analysis using the Monte Carlo Simulation (MCS) method. The optimization problem at hand is a multicriteria optimization problem. Evolutionary Algorithms, and in particular Evolution Strategies, are employed. Each design is checked whether it satisfies the provisions of European design codes (Eurocodes 3 and 8) with a prescribed probability of violation.

ROBUST DESIGN STRUCTURAL OPTIMIZATION

In the present study the robust design versus the deterministic based sizing optimization of large-scale space trusses is investigated. The robustness of the constraints is also considered using the overall probabilities of violation of the structural constraints, as a result of the variation of the random structural parameters. The random variables chosen are the cross-sectional dimensions of structural members the material properties modulus of elasticity *E*, the yield stress σ_y and the lateral loads.

Deterministic based optimization (DBO)

In the deterministic sizing optimization problems the aim is to minimize the weight of the structure under certain deterministic behavioral constraints usually on stresses and displacements. In Robust Design Optimization additional probabilistic objectives are considered. A discrete DBO problem can be formulated in the following form

$$\begin{array}{ll} \min & F(s) \\ \text{subject to} & g_j(s) \leq 0 \quad j = 1,\dots,k \\ & s_i \in R^d, \quad i = 1,\dots,n \end{array} \tag{1}$$

where *F(s)* is the objective function, *s* is the vector of geometric design variables, which can take values only from a discrete given set R^d, and $g_j(s)$ are the deterministic constraints. Most frequently the deterministic constraints refer to the member stresses and nodal displacements or the inter-storey drifts. In this study three types of constraints are imposed to the sizing optimization problem: (i) stress (ii) compression force (for buckling) and (iii) displacement constraints. The stress constraint can be written as follows

$$\begin{array}{l} \sigma_{max} \leq \sigma_a \\ \sigma_a = \dfrac{\sigma_y}{1.10} \end{array} \tag{2}$$

where σ_{max} is the maximum axial stress in each element group for all loading cases, σ_a is the allowable axial stress according to Eurocode 3 [9] and σ_y is the yield stress. For members under compression an additional constraint is used

$$\begin{aligned} &\left|P_{c,max}\right| \leq P_{cc} \\ &P_{cc} = \frac{P_e}{1.05} \\ &P_e = \frac{\pi^2 EI}{L_{eff}^2} \end{aligned} \tag{3}$$

where $P_{c,max}$ is the maximum axial compression force for all loading cases, P_e is the critical Euler buckling force in compression, taken as the first buckling mode of a pin-connected member, and L_{eff} is the effective length. The effective length is taken equal to the actual length. Similarly, the displacement constraints can be written as

$$\left|d\right| \leq d_a \tag{4}$$

where d_a is the limit value of the displacement at a certain node or the maximum nodal displacement. A constraint of 200mm on the maximum deflection is imposed.

Robust design optimization (RDO)

In a robust design sizing optimization problem an additional objective function is considered which is related to the influence of the random nature of some structural parameters on the response of the structure. In the present study the aim is to minimize both the weight and the variance of the response of the structure. The constraint functions are also varied due to variations of the random structural parameters. An optimum solution in DBO might violate some of the constraints for some values of the random structural parameters. In the formulation of the RDO considered in this study the variance of the constraints has been also taken into account and additional constraint functions of stochastic nature are considered. The mathematical formulation of the RDO problem implemented in this study is as follows

$$\begin{aligned} \text{min} \quad & \Phi(s) \\ \text{subject to} \quad & g_j(s) \leq 0 \quad j = 1,...,k \\ & p_{v,j} \leq p_{all} \quad j = 1,...,k \\ & s_i \in R^d, \quad i = 1,...,n \end{aligned} \tag{5}$$

where $\Phi(s)$ is the multi-objective function, s is the vector of geometric design variables, which can take values only from the given discrete set R^d, $g_j(s)$ are the deterministic constraints while $p_{v,j}$ is the probability of violation of the j-th deterministic constraint bound by an upper allowable probability equal to p_{all}. The multi-objective function is expressed as

$$\Phi(s) = wF(s) + (1-w)\sigma u \tag{6}$$

where $F(s)$ is the weight of the structure and σu is the variance of the response of the structure. The proposed robust design sizing optimization methodology proceeds with the following steps:

1. At the outset of the optimization procedure the geometry, the boundaries and the reference loads of the structure under investigation are defined.
2. The constraints are defined in order for the optimization problem to be formulated as in eq. (5).
3. The optimization phase is carried out with ES where feasible designs are produced at each generation. The feasibility of the designs is checked for each design vector with respect to both deterministic and probabilistic constraints of the problem.
4. The satisfaction of the deterministic constraints is monitored through a finite element analysis of the structure.

5. Stochastic analysis of the structure using the MCS technique is carried out in order to evaluate the probability of violation of the structural constraints and to calculate the variance of the response of the structure.
6. If the convergence criteria for the optimization algorithm are satisfied then the optimum solution has been found and the process is terminated, else the whole process is repeated from step 3 with a new generation of design vectors.

Probabilistic constraints define the feasible region of the design space by restricting the probability that a deterministic constraint is violated within the allowable probability of violation. The probabilistic constraints that are employed in this study enforce the condition that the probabilities of violation of the structure are smaller than a certain value.

MONTE CARLO SIMULATION

In stochastic analysis of structures the MCS method is particularly applicable when an analytical solution is not attainable. This is mainly the case in problems of complex nature with a large number of basic random variables (random structural parameters), where all other stochastic analysis methods are not applicable. Despite the fact that the mathematical formulation of the MCS is relatively simple and the method has the capability of handling practically every possible case regardless of its complexity, this approach has not received an overwhelming acceptance due to the excessive computational effort that it requires. Furthermore, soft computing methodologies and parallel processing have been recently implemented having a beneficial effect on the efficiency of MCS [11]. In the current study the MCS has been employed for the calculation of the probability of violation of the behavioral constraints and the variance of the response of the structure due to the random nature of some structural parameters. Both probability of violation and the variance of the response of the structure are required in the framework of an RDO problem.

In structural stochastic analysis problems where the probability of violation of some behavioral constraints is to be calculated, MCS can be stated as follows: Expressing the limit state function as $G(x)<0$, where $x=(x_1,x_2,...,x_M)$ is the vector of the random structural parameters, the probability of violation of the behavioral constraints can be written as

$$p_{viol} = \int_{G(x)\geq 0} f_x(x)dx \tag{7}$$

where $f_x(x)$ denotes the joint probability of violation for all random structural parameters. Since MCS is based on the theory of large numbers (N_∞) an unbiased estimator of the probability of violation is given by

$$p_{viol} = \frac{1}{N_\infty}\sum_{j=1}^{N_\infty} I(x_j) \tag{8}$$

in which $I(x_j)$ is an indicator for successful and unsuccessful simulations defined as

$$I(x_j) = \begin{cases} 1 & \text{if } \mathrm{G(x_j)} \geq 0 \\ 0 & \text{if } \mathrm{G(x_j)} < 0 \end{cases} \tag{9}$$

In order to estimate p_{viol} an adequate number of N independent random samples is produced using a specific, uniform probability density function of the vector x. The value of the violation function is computed for each random sample x_j and the Monte Carlo estimation of p_{viol} is given in terms of sample mean by

$$p_{viol} \cong \frac{\mathrm{N_H}}{N} \tag{10}$$

where N_H is the number of successful simulations and N the total number of simulations.

MULTIPLE OBJECTIVE OPTIMIZATION

In formulating an optimization problem the choice of the design variables, criteria and constraints represents undoubtedly the most important decision to be made by the engineer. In general, the mathematical formulation of a multi-objective problem includes a set of n design variables, a set of m objective functions and a set of k constraint functions and can be defined as follows

$$\begin{aligned} &\min_{s \in F} && [f_1(s), f_2(s), \ldots, f_m(s)]^T \\ &\text{subject to} && g_j(s) \leq 0 \quad j = 1,\ldots,k \\ & && s_i \in R^d, \quad i = 1,\ldots,n \end{aligned} \tag{11}$$

where the vector $s = [s_1\, s_2\, \ldots\, s_n]^T$ represents a design variable vector and F is the feasible set in design space R^n which is defined as the set of design variables that satisfy the constraint functions $g(s)$ in the form:

$$F = \{s \in R^n \mid g_j(s) \leq 0 \quad j = 1,\ldots,k\} \tag{12}$$

Usually there exists no unique point which would give an optimum for all m criteria simultaneously. Thus the common optimality condition used in single-objective optimization must be replaced by a new concept, the so called Pareto optimum: A design vector $s^* \in F$ is Pareto optimal for the problem of eq. (11) if and only if there is no other design vector $s \in F$ such that:

$$\begin{aligned} &f_i(s) \leq f_i(s^*) \text{ for } i = 1,\ldots,m \\ &\text{with } f_i(s) < f_i(s^*) \text{ for at least one objective } i \end{aligned} \tag{13}$$

The solutions of optimization problems with multiple objectives constitute the set of the Pareto optimum solutions. The problem of eq. (11) can be regarded as being solved after the set of Pareto optimal solutions has been determined. In practical applications, however, the designer seeks for a unique final solution. Thus a compromise should be made among the available Pareto optimal solutions.

Linear weighting method

The first method, called the linear weighting method, combines all the objectives into a single scalar parameterized objective function by using weighting coefficients. If w_i, $i=1,2,\ldots,m$ are the weighting coefficients, the problem of eq. (5) can be written as follows:

$$\min_{s \in F} \sum_{i=1}^{m} w_i\, f_i(s) \tag{14}$$

with no loss of generality the following normalization of the weighting coefficients is employed:

$$\sum_{i=1}^{m} w_i = 1 \tag{15}$$

By varying these weights it is now possible to generate the set of Pareto optimum solutions for the problem of eq. (11). The values of the weighting coefficients are adjusted according to the importance of each criterion. Every combination of those weighting coefficients correspond to a single Pareto optimal solution, thus, performing a set of optimization processes using different weighting coefficients it is possible to generate the full set of the Pareto optimal solutions.

Evolution Strategies for structural multi-objective optimization problems

The application of evolutionary algorithms in multi-objective optimization problems has attracted the interest of a number of researchers in the last ten years due to the difficulty of conventional optimization techniques, such as gradient based methods, to be extended to multi-objective optimization problems. EA, however, have been recognized to be more appropriate to multi-objective optimization problems since early in their development [12,13]. Multiple individuals can search for multiple solutions simultaneously, taking advantage of any similarities available in the family of possible solutions to the problem.

In our implementation, where the weighting method is used, in order to generate a set of Pareto optimal solutions, the optimization procedure initiates with a set of parent design vectors needed by the ES optimizer and a set of weighting coefficients for the combination of all objectives into a single scalar parameterized objective function. These weighting coefficients are not set by the designer but are being systematically varied by the optimizer after a Pareto optimal solution has been achieved. There is an outer loop which systematically varies the parameters of the parameterized objective function, and is called decision making loop. The inner loop is the classical ES process, starting with a set of parent vectors. If any of these parent vectors gives an infeasible design then this parent vector is modified until it becomes feasible. Subsequently, the offsprings are generated and checked whether they are in the feasible region. According to the $(\mu+\lambda)$ selection scheme in every generation the values of the objective function of the parent and the offspring vectors are compared and the worst vectors are rejected, while the remaining ones are considered to be the parent vectors of the new generation. On the other hand, according to the (μ,λ) selection scheme only the offspring vectors of each generation are used to produce the new generation. This procedure is repeated until the chosen termination criterion is satisfied. The number of parents and offsprings involved affects the computational efficiency of the multi-membered ES scheme discussed in this work. It has been observed that when the values of μ and λ are equal to the number of the design variables, better results are produced.

The ES algorithm combined with the standard methods can be stated as follows:

Outer loop - Decision making loop
Set the parameters w_i of the parameterized objective function
Inner loop - ES loop

1. *Selection step :* selection of s_i (i = 1,2,...,μ) parent vectors of the design variables
2. *Analysis step*
3. *Evaluation of parameterized objective function*
4. *Constraints check :* all parent vectors become feasible
5. *Offspring generation :* generate s_j, (j=1,2,...,λ) offspring vectors of the design variables
6. *Analysis step*
7. *Evaluation of the parameterized objective function*
8. *Constraints check :* if satisfied continue, else change s_j and go to *step 5*
9. *Selection step :* selection of the next generation parents according to (μ+λ) or (μ,λ) selection schemes
10. *Convergence check :* If satisfied stop, else go to *step 5*

End of Inner loop
End of Outer loop

NUMERICAL RESULTS

A three dimensional 39-bar truss shown in Figure 1 is considered for presenting the efficiency of the proposed RDO methodology. The height of the structure is 16 m (Figure 1b), while its basis is an equilateral triangle of side 6.93 m (Figure 1c). Two objective functions are used, the weight and the variance of the response of the structure, under the constraints on stresses and displacements imposed by the design codes [9,10]. Due to engineering practice demands, the members are divided into groups having the same design variables. This linking of elements results in a trade-off between the use of more material and the need of symmetry and uniformity of structures due to practical considerations. Furthermore, it has to be taken into account that due to manufacturing limitations the design variables are not continuous but discrete since cross-sections belong to a certain pre-defined set provided by the manufacturers. Thus the design variables considered are the dimensions of the members of the structure, four groups in total, taken from the Circular Hollow Section (CHS) table of the Eurocode. For each design variable, two stochastic variables are assigned: The external diameter d and the thickness t of the circular hollow section. A vertical load V=2kN is applied to all nodes, while a probabilistic horizontal load F of mean value 8 kN is applied to the top nodes at the x-direction.

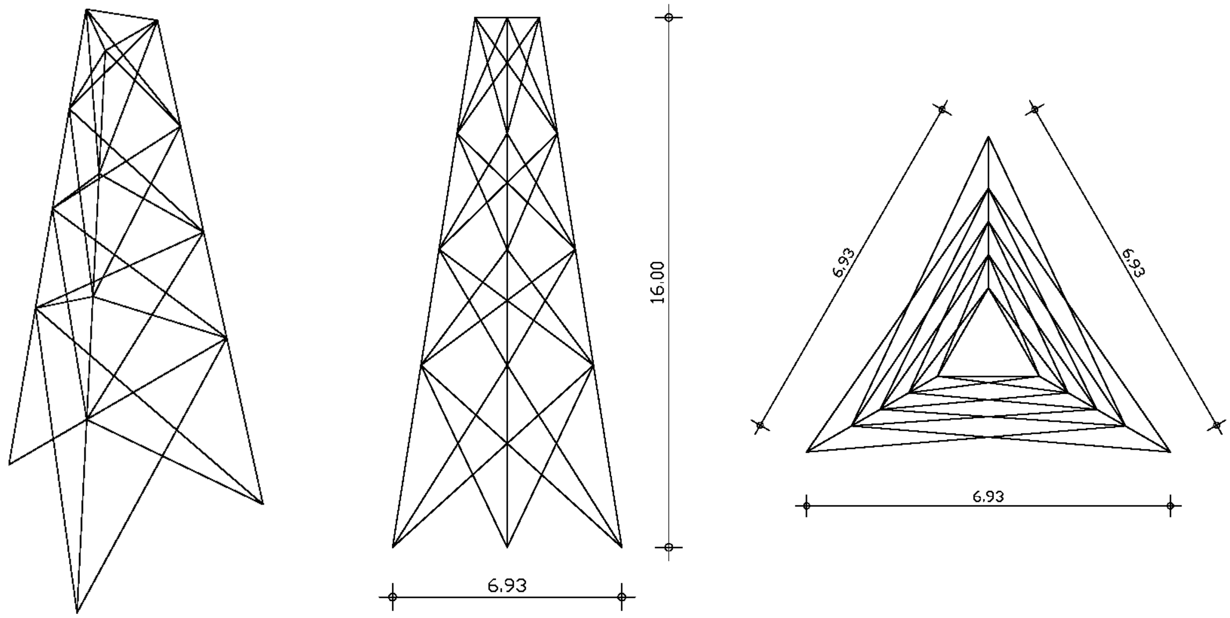

Fig. 1 Three Dimensional 39-bar truss example (a) 3D view, (b) Side view, (c) Top view

The types of probability density functions, the mean values, and the variances of the random parameters are shown in Table 1. For this test case the (μ+λ)-ES approach is used with μ=λ=5, while a sample size of 1,000 simulations is taken for the MCS.

Table 1: Characteristics of the random variables

		Probability Density Function	**Mean value μ**	**Standard Deviation σ**	**σ/μ**	**95% of values within**
E (kN/m^2)	Young's Modulus	Normal	2.10E+08	1.50E+07	7.14%	(1.81E+08, 2.39E+08)
σ_y (kN/m^2)	Allowable stress	Normal	355000	35500	10.00%	(2.85E+05, 4.25E+05)
F (kN)	Horizontal loading	Normal	8	3	37.50%	(2.12, 13.88)
d	CHS Diameter	Normal	d_i *	0.02 d_i	2%	(0.9608 d_i, 1.0392 d_i)
t	CHS Thickness	Normal	t_i *	0.02 t_i	2%	(0.9608 t_i, 1.0392 t_i)

* Taken from the Circular Hollow Section (CHS) table of the Eurocode, for every design

The resultant Pareto front curve is depicted in Figure 2, with the weight of the structure and the standard deviation of the horizontal displacement on the horizontal and vertical axis, respectively. The Pareto front curve shows a strong contradiction between the two objective functions in question.

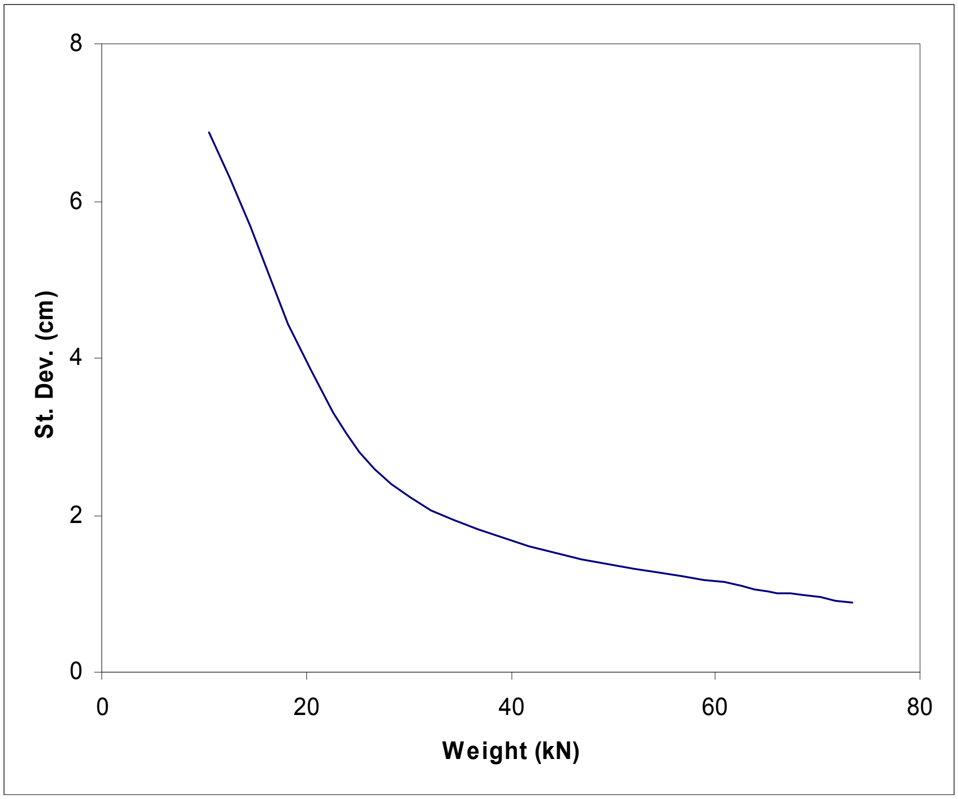

Fig. 2 Pareto front curve

CONCLUDING REMARKS

Evolution Strategies can be considered as an efficient tool for multi-objective design optimization of structural problems and in particular for the robust design sizing optimization problem. The proposed two stages evolution strategies method for treating multi-objective optimization problems proved to be a robust and reliable optimization tool.

The deterministic based formulation of this structural optimization problem would converge to an optimum solution with the minimum weight, yet the resultant structural response would vary widely, and consequently the quality of the final design would be in doubt. In order to account for the randomness of parameters that affect the response of the structure, an RDO formulation of the optimization problem has to be used.

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