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Structural Optimization Considering the Probabilistic System Response

M. Papadrakakis, N. D. Lagaros and V. Plevris

Institute of Structural Analysis & Seismic Research
National Technical University of Athens
Zografou Campus, Athens 15780, Greece

e-mail: mpapadra@central.ntua.gr

Abstract In engineering problems, the randomness and uncertainties are inherent and the scatter of structural parameters from their nominal ideal values is unavoidable. In the case of Reliability Based Design Optimization (RBDO) and Robust Design Optimization (RDO) the uncertainties play a dominant role in the structural optimization problem. In an RBDO problem additional non deterministic constraint functions are considered while RDO yields a design with a state of robustness, so that its performance is the least sensitive to the variability of the uncertain variables. The first part examines the application of Neural Networks to the RBDO of large scale structural systems. In the second part an RDO structural problem is considered. The task of robust design optimization of structures is formulated as a multi-criteria optimization problem, in which the design variables of the optimization problem, together with other design parameters such as the modulus of elasticity and the yield stress are considered as random variables with a mean value equal to their nominal value.

Keywords: Structural optimization, reliability analysis, robust design, Evolution Strategies, Monte Carlo simulation, Neural Networks.

1. Introduction

In recent years, probabilistic based formulations of optimization problems have been developed to account for uncertainty and randomness through stochastic simulation and probabilistic analysis. Stochastic analysis methods have been developed over the last two decades [1, 2] and have stimulated the interest for the probabilistic optimum design of structures. There are two distinguished design formulations that account for probabilistic systems response: Robust Design Optimization (RDO) [3-5] and Reliability-Based Design Optimization (RBDO) [6-8]. RDO methods primarily seek to minimize the influence of stochastic variations on the mean design, and traditionally rely on rough approximations of the stochastic response about the mean design, such as the First-Order Second Moment methods. On the other hand, the main goal of RBDO methods is to design for safety with respect to extreme events and generally require a stochastic analysis of the system response far off the mean design such as Monte Carlo simulation or reliability methods.

Despite the theoretical advancements in the field of reliability analysis, serious computational obstacles arise when treating realistic problems. In particular, the reliability-based design optimization of large-scale structural systems is an extremely computationally intensive task, as shown by Tsompanakis and Papadrakakis [3]. Despite the improvements achieved in the efficiency of the computational methods for treating reliability analysis problems, they still require disproportionate computational effort for practical reliability problems. This is the reason why very few successful numerical investigations are known in the field [6-8].

In the first part of the present study the reliability-based sizing optimization of multi-storey space frames is investigated. The objective function is the weight of the structure while the constraints are both deterministic (stress and displacement limitations) and probabilistic (the overall probability of failure of the structure). Randomness of loads, material properties, and member geometry are taken into account in the reliability analysis using the Monte Carlo simulation (MCS) method. The probability of failure of a frame structure is determined via a limit state elasto-plastic analysis. In this work two methodologies that combine Evolution Strategies and Neural Networks (ES-NN) are examined. In the first, a trained NN is applied to predict the response of the structure in terms of deterministic and probabilistic constraints checks due to different sets of design variables. In the second methodology, the limit state elasto-plastic analyses required during the MCS method are replaced by the NN predictions of the structural behaviour up to collapse. For every MCS that is required in order to perform the probabilistic constraints check, an NN is trained that utilizes available information generated from selected conventional elasto-plastic analyses. The trained NN is used to predict the critical load factor due to different sets of basic random variables.

In the second part of the study the robust design sizing optimization of large-scale space trusses is investigated. The objective functions considered are the weight and the variance of the response of the structure, subject to stress and displacement constraints imposed by the design codes [10, 11]. Randomness of loads, material properties, and member geometry are taken into account in the stochastic analysis using the MCS method. The optimization problem at hand is a multicriteria optimization problem. Evolutionary Algorithms, and in particular Evolution Strategies, are employed. Each design is checked whether it satisfies the provisions of the European design codes (Eurocodes 3 and 8) with a prescribed probability of violation.

2. Monte Carlo Simulation

In stochastic analysis of structures, the MCS method is particularly applicable when an analytical solution is not attainable. This is mainly the case in problems of complex nature with a large number of basic random variables (random structural parameters), where all other stochastic analysis methods cannot be not applicable. Despite the fact that the mathematical formulation of the MCS method is relatively simple and the method has the capability of handling practically every possible case regardless of its complexity, this approach has not received an overwhelming acceptance due to the excessive computational effort that it requires. Furthermore, soft computing methodologies and parallel processing have been recently implemented having a beneficial effect on the efficiency of the method [12]. In the current study the MCS method has been employed for the calculation of the probability of failure, the probability of violation of the behavioral constraints and the variance of the response of the structure due to the random nature of some structural parameters.

In structural stochastic analysis problems where the probability of violation of some behavioral constraints is to be calculated, the MCS method can be stated as follows: Expressing the limit state function as $G(x)<0$, where $x=(x_1,x_2,...,x_M)$ is the vector of the random structural parameters, the probability of violation of the behavioral constraints can be written as

$$p_{viol} = \int_{G(x)\geq 0} f_x(x)dx \quad (1)$$

where $f_x(x)$ denotes the joint probability of violation for all random structural parameters. Since MCS is based on the theory of large numbers (N_∞) an unbiased estimator of the probability of violation is given by

$$p_{viol} = \frac{1}{N_\infty}\sum_{j=1}^{N_\infty} I(x_j) \quad (2)$$

in which $I(x_j)$ is an indicator for successful and unsuccessful simulations defined as

$$I(x_j) = \begin{cases} 1 & \text{if } \ G(x_j) \geq 0 \\ 0 & \text{if } \ G(x_j) < 0 \end{cases} \quad (3)$$

In order for p_{viol} to be estimated, an adequate number of N independent random samples is produced using a specific probability density function of the vector x. The value of the violation function is computed for each random sample x_j and the Monte Carlo estimation of p_{viol} is given in terms of sample mean by

$$p_{viol} \cong \frac{N_H}{N} \quad (4)$$

where N_H is the number of successful simulations and N the total number of simulations.

3. Structural Optimization

A structural optimization problem can be classified with respect to the type of the structural behaviour, the type of design variables and the type of the structure to be optimized. There are mainly three classes of structural optimization problems: Sizing, shape and topology (or layout), depending on the type of the structure to be optimized. An optimization problem is characterized as deterministic or probabilistic depending on the consideration or not of the uncertainties involved in the structural behaviour. It can also be classified as discrete or continuous, depending on the type of the design variables. In deterministic sizing optimization problems the aim is to minimize the weight of the structure under certain deterministic behavioral constraints, usually on stresses and displacements. In probabilistic sizing optimization problems, the randomness and the uncertainties that are inherent in engineering problems, have to be taken into consideration.

Due to engineering practice demands, the members of a frame or truss structure are divided into groups, with the members of each group sharing the same design variables. This linking of elements results in a trade-off between the use of more material and the need of symmetry and uniformity of structures due to practical considerations. Furthermore, it has to be taken into account that in most cases, due to manufacturing limitations the design variables cannot be considered as continuous but discrete since cross-sections belong to a certain set provided by the manufacturers.

3.1 Deterministic Based Optimization (DBO)

In deterministic sizing optimization problems the aim is to minimize the weight of the structure under certain deterministic behavioral constraints usually on stresses and displacements. A discrete DBO problem can be formulated in the following form

$$\begin{aligned} &\min && F(s) \\ &\text{subject to} && g_j(s) \le 0 \quad j = 1,...,k \\ &&& s_i \in R^d, \quad i = 1,...,n \end{aligned} \tag{5}$$

where *F(s)* is the objective function, s is the vector of geometric design variables, which can take values only from a discrete given set R^d, and $g_j(s)$ are the deterministic constraints. Most frequently the deterministic constraints refer to member stresses and nodal displacements or the inter-storey drifts. In this study three types of constraints are imposed to the sizing optimization problem: (i) stress (ii) compression force (for buckling) and (iii) displacement constraints.

3.1.1 Frame structures

The stress constraint function for beams subjected to biaxial bending under compression is given by the formula:

$$\frac{N_{sd}}{Af_y / \gamma_{M1}} + \frac{M_{sd,y}}{W_{pl,y} f_y / \gamma_{M1}} + \frac{M_{sd,z}}{W_{pl,z} f_y / \gamma_{M1}} \le 1.0 \tag{6}$$

where N_{sd}, $M_{sd,y}$, $M_{sd,z}$ are the computed stress resultants, $W_{pl,y}$, $W_{pl,z}$ are the plastic first moments of inertia, f_y is the yield stress and γ_{M1} is a safety factor equal to 1.10 [10]. The interstorey drift constraint employed in a frame structure can be written as:

$$\frac{d_r}{\nu} \le 0.006 \times h \tag{7}$$

where ν is a reduction factor for the serviceability limit state (taken equal to 2.5 for the test example considered in this study) and d_r is the relative drift between two consecutive stories.

3.1.2 Truss structures

The stress constraints considered in a truss sizing optimization problem can be written as follows

$$\begin{aligned} &\sigma_{max} \le \sigma_a \\ &\sigma_a = \frac{\sigma_y}{1.10} \end{aligned} \tag{8}$$

where σ_{max} is the maximum axial stress in each element group for all loading cases, σ_a is the allowable axial stress according to Eurocode 3 [10] and σ_y is the yield stress. For members under compression an additional constraint is implemented

$$\begin{aligned} &\left| P_{c,max} \right| \le P_{cc} \\ &P_{cc} = \frac{P_e}{1.05} \\ &P_e = \frac{\pi^2 EI}{L_{eff}^2} \end{aligned} \tag{9}$$

where $P_{c,max}$ is the maximum axial compression force for all loading cases, P_e is the critical Euler buckling force in compression, taken as the first buckling mode of a pin-connected member, and L_{eff} is the effective length. The effective length is taken equal to the actual length. Similarly, the displacement constraints can be written as

$$|d| \leq d_a \tag{10}$$

where d_a ais the limit value of the displacement at a certain node or the maximum nodal displacement. A constraint of 200mm on the maximum deflection is imposed.

3.2 Reliability Based Design Optimization (RBDO)

In deterministic sizing optimization problems the aim is to minimize the weight of the structure under certain deterministic behavioral constraints usually on stresses and displacements. In RBDO problems additional probabilistic constraints are imposed in order for various random parameters to be taken into account. Probabilistic constraints define the feasible region of the design space by restricting the probability that a deterministic constraint is violated within the allowable probability of violation. The probabilistic constraint that is employed in this study enforces the condition that the probability failure of the structure is smaller than a certain specified value.

In the present study the reliability-based sizing optimization of large-scale multi-storey 3-D frames is investigated. Thus the overall probability of failure of the structure, as a result of a limit state elasto-plastic analysis, is taken as the global reliability constraint. The probabilistic design variables are chosen to be the cross-sectional dimensions of the structural members and the material properties, modulus of elasticity E and yield stress σ_y.

A discrete RBDO problem can be formulated in the following form

$$\begin{array}{lll} \min & F(s) & \\ \text{subject to} & g_j(s) \leq 0 & j=1,...,m \\ & s_i \in R^d, & i=1,...,n \\ & p_f \leq p_a & \end{array} \tag{11}$$

F(s) is the objective function, *s* is the vector of geometric design variables, which can take values only from the given discrete set R^d, $g_j(s)$ are the deterministic constraints and p_f is the probability of failure of the structure - required to remain below a threshold value (p_a) that comprises the probabilistic constraint. Most frequently the deterministic constraints of the structure are the member stresses and the nodal displacements or the inter-storey drifts.

The proposed RBDO sizing optimization methodology proceeds with the following steps:

1. At the outset of the optimization procedure the geometry, boundaries and reference loads of the structure under investigation are defined.
2. The constraints are defined in order for the optimization problem to be formulated, as in eq. (10).
3. The optimization phase is carried out with Evolution Strategies where feasible designs are produced at each generation. The feasibility of each design vector is checked with respect to both the deterministic and the probabilistic constraints of the problem.
4. The satisfaction of the deterministic constraints is monitored through a finite element analysis of the structure.
5. The satisfaction of the probabilistic constraints is realized with a reliability analysis of the structure using the MCS technique for the evaluation of the probability of failure.
6. If the convergence criteria for the optimization algorithm are satisfied, then the optimum solution has been achieved and the process is terminated, otherwise the whole process is repeated from step 3 with a new generation of design vectors.

In this work the reliability constraint is related to the ultimate load-carrying capacity of the space frame structure. This failure criterion is considered to be the formation of a mechanism as a result of a limit state elasto-plastic analysis of the structure without considering member instability effects. The adopted incremental non-holonomic first order step-by-step limit state analysis is based on the generalized plastic node concept [13, 14]. The non-linear yield surface is approximated by a multi-faceted surface thus avoiding iterations at each load step. In order to prevent the occurrence of very small load steps, a second internal and homothetic to the initial yield surface is implemented which forms a plastic zone for the activation of the plastic nodes [15].

3.3 Robust Design Optimization (RDO)

In the present study the robust design versus the deterministic based sizing optimization of large-scale space trusses is investigated. The robustness of the constraints is considered using the overall probabilities of violation of the structural constraints, as a result of the variation of the random structural parameters. The random variables chosen are the cross-sectional dimensions of the structural members, the material properties modulus of elasticity E and yield stress σ_y and the lateral loads.

In a robust design sizing optimization problem an additional objective function is considered which is related to the influence of the random nature of some structural parameters on the response of the structure. In the present study the aim is to minimize both the weight and the variance of the response of the structure. The constraint functions are also varied due to variations of the random structural parameters. An optimum solution in deterministic-based design optimization might violate some of the constraints for some values of the random structural parameters. In the formulation of the RDO problem considered in this study the variance of the constraints has also been taken into account and additional constraint functions of stochastic nature are considered. The mathematical formulation of a discrete RDO problem, as implemented in this study is as follows

$$\begin{aligned} &\min && \Phi(s) \\ &\text{subject to} && g_j(s) \le 0 \quad j = 1,...,k \\ & && p_{v,j} \le p_{all} \quad j = 1,...,k \\ & && s_i \in R^d, \quad i = 1,...,n \end{aligned} \tag{12}$$

where $\Phi(s)$ is the multi-objective function, s is the vector of geometric design variables, which can take values only from the given discrete set R^d, $g_j(s)$ are the deterministic constraints while $p_{v,j}$ is the probability of violation of the j-th deterministic constraint bound by an upper allowable probability equal to p_{all}. The multi-objective function is expressed as

$$\Phi(s) = wF(s) + (1 - w)\sigma_u \tag{13}$$

where $F(s)$ is the weight of the structure and σ_u is the variance of the response of the structure. The proposed robust design sizing optimization methodology proceeds with the following steps:

1. At the outset of the optimization procedure the geometry, the boundaries and the reference loads of the structure under investigation are defined.
2. The constraints are defined in order for the optimization problem to be formulated as in eq. (12).
3. The optimization phase is carried out with ES where feasible designs are produced at each generation. The feasibility of the designs is checked for each design vector with respect to both deterministic and probabilistic constraints of the problem.
4. The satisfaction of the deterministic constraints is monitored through a finite element analysis of the structure.
5. Stochastic analysis of the structure using the MCS technique is carried out in order to evaluate the probability of violation of the structural constraints and to calculate the variance of the response of the structure.
6. If the convergence criteria for the optimization algorithm are satisfied, then the optimum solution has been achieved and the process is terminated, otherwise the whole process is repeated from step 3 with a new generation of design vectors.

Probabilistic constraints define the feasible region of the design space by restricting the probability that a deterministic constraint is violated within the allowable probability of violation. The probabilistic constraints that are employed in this study enforce the condition that the probabilities of violation of the structure are smaller than a certain value.

4. Evolution Strategies (ES)

The first version of the Evolution Strategies (ES) method was based on a population consisting of one individual only. The two membered ES scheme is the minimal concept for an imitation of organic evolution.

The two principles of mutation and selection, which Darwin in 1859 recognised to be the most important, are taken as rules for variation of the parameters and for recursion of the iteration sequence respectively. The multi-membered Evolution Strategies employed in this study differ from the previous two-membered strategies in the size of the population.

4.1 ES in structural optimization problems

In structural optimization problems, where both the objective and the constraints can be highly non-linear functions of the design variables, the computational effort spent in gradient calculations needed for mathematical programming algorithms is usually high. In a recent study by Papadrakakis et al. [16] it was found that probabilistic search methods in structural optimization are computationally efficient compared to gradient-based optimization methods, even if large number of optimization steps are needed to reach the optimum. These optimization steps are computationally less expensive than those of mathematical programming algorithms as they do not need gradient information. This property of probabilistic search methods is of great importance in the case of RBDO and RDO problems, since the calculation of the derivatives of the probabilistic constraints can be extremely time-consuming. Furthermore, probabilistic methodologies are more capable of finding the global optimum due to their random search, whereas mathematical programming algorithms may be trapped in local optima.

The ES optimization procedure starts with an initial set of parent vectors. If any vector of the parent set corresponds to an infeasible design then it is modified until it becomes feasible. Subsequently, the offspring design vectors are generated and checked whether they are in the feasible region. According to the $(\mu+\lambda)$ selection scheme the values of the objective function of the parent and the offspring vectors in every generation are compared; the worst vectors are rejected, while the best ones are considered as the parent vectors of the new generation. This procedure is repeated until the chosen termination criterion is satisfied. The ES algorithm for structural optimization applications can be stated as follows:

1. *Selection step:* Selection of s_i ($i = 1,2,...,\mu$) parent design vectors.
2. *Analysis step:* Solve $K(s_i)x_i = b$ ($i=1,2,...,\mu$).
3. *Constraints check:* If satisfied continue, else change s_j and go to *step 1.*
4. *Offspring generation:* Generate s_j, ($j=1,2,...,\lambda$) offspring design vectors.
5. *Analysis step:* Solve $K(s_j)x_j = b$ ($j=1,2,...,\lambda$).
6. *Constraints check:* If satisfied continue, else change s_j and go to *step 4.*
7. *Selection step:* Selection of the next generation parent design vectors.
8. *Convergence check:* If satisfied stop, else go to *step 4.*

An important characteristic of the ES method, that distinguishes it from most conventional optimization algorithms, is that instead of using a single design point it works simultaneously with a population of design points. This allows the natural implementation of the ES optimization procedure in parallel computing environments where the finite element analyses of the members of each population are performed independently and concurrently.

4.2 Reliability-based structural optimization using MCS, ES and NN

In reliability analysis of elasto-plastic structures using MCS the computed critical load factors are compared to the corresponding external loading leading to the computation of the probability of structural failure. The probabilistic constraints enforce the condition that the probability of a local failure of the system or the global system failure is smaller than a certain value (i.e. 10^{-5} - 10^{-3}). In this work the overall probability of failure of the structure, as a result of limit state elasto-plastic analyses, is taken as the global reliability constraint. The probabilistic design variables are chosen to be the cross-sectional dimensions of the structural members and the material properties (E, σ_y).

MCS requires a number of limit state elasto-plastic analyses that can be dealt with independently and concurrently. This allows the natural implementation of the MCS method in parallel computing environment as well. The most straightforward parallel implementation of the MCS method is to assign one limit state elasto-plastic analysis to a processor without any need of inter-processor communication during the analysis phase.

4.2.1 NN used for deterministic and probabilistic constraints check

In this methodology, a trained NN that utilizes information generated from a number of properly selected design vectors is used to perform both the deterministic and probabilistic constraints checks that are needed during the optimization process. After the selection of the suitable NN architecture, the training procedure is performed using a number (M) of data sets in order to obtain the I/O pairs needed for the NN training. The trained NN is then applied to predict the response of the structure in terms of deterministic and probabilistic constraints checks due to different sets of design variables.

The combined ES-NN optimization procedure is performed in two phases. The first phase includes the training set selection, the corresponding structural analysis and MCS for each training set required to obtain the necessary I/O data for the NN training, and finally the training and testing of a suitable NN configuration. The second phase is the ES optimization stage where the trained NN is used to predict the response of the structure in terms of the deterministic and probabilistic constraints checks due to different sets of design variables.

This ES-NN methodology can be described with the following algorithm 1:

- NN training phase:
  1. Training set selection step: Select M input patterns.
  2. Deterministic constraints check: Perform the check for each input pattern vector.
  3. Monte Carlo Simulation step: Perform MCS for each input pattern vector.
  4. Probabilistic constraints check: Perform the check for each input pattern vector.
  5. Training step: Training of the NN.
  6. Testing step: Test the trained NN.
- ES-NN optimization phase:
  1. Parents Initialization.
  2. NN (Deterministic-Probabilistic) constraints check: All parents become feasible.
  3. Offspring generation.
  4. NN (Deterministic-Probabilistic) constraints check: If satisfied continue, else go to step 3.
  5. Parents' selection step.
  6. Convergence check.

4.2.2 NN prediction of the critical load in structural failure

In the second methodology the limit state elasto-plastic analyses required during the MCS are replaced by the NN prediction of the structural behavior up to collapse. For every MCS an NN is trained utilizing available information generated from selected conventional elasto-plastic analyses. The limit state analysis data is processed to obtain input and output pairs, which are used for the NN training. The trained NN is then used to predict the critical load factor due to different sets of basic random variables.

At each ES cycle (generation) a number of MCS is carried out. In order to replace the time consuming limit state elasto-plastic analyses by predicted results obtained with a trained NN, a training procedure is performed based on the data collected from a number of conventional limit state elasto-plastic analyses. After the training phase is concluded the trained NN predictions replace the conventional limit state elasto-plastic analyses, for the current design. For the selection of the suitable training pairs, the sample space for each random variable is divided into equally spaced distances. The central points within the intervals are used as inputs for the limit state analyses.

This ES-NN methodology can be described with the following algorithm 2:

1. Parents Initialization.
2. Deterministic constraints check: All parents become feasible.
3. Monte Carlo Simulation step:
   3a. Selection of the NN training set.
   3b. NN training for the limit load.
   3c. NN testing.
   3d. Perform MCS using NN.
4. Probabilistic constraints check: All parents become feasible.
5. Offspring generation.
6. Deterministic constraints check: If satisfied continue, else go to step 5.
7. Monte Carlo Simulation step:
   7a. Selection of the NN training set.
   7b. NN training for the limit load.
   7c. NN testing.
   7d. Perform MCS using NN.

8. Probabilistic constraints check: If satisfied continue, else go to step 5.
9. Parents' selection step.
10. Convergence check.

5. Multiple Objective Optimization

In formulating an optimization problem the choice of the design variables, criteria and constraints represents undoubtedly the most important decision to be made by the engineer. In general, the mathematical formulation of a multi-objective problem that includes a set of n design variables, a set of m objective functions and a set of k constraint functions can be defined as follows

$$\begin{aligned} &\min_{s\in F} && [f_1(s), f_2(s), \ldots, f_m(s)]^T \\ &\text{subject to} && g_j(s) \le 0 \quad j = 1,\ldots,k \\ & && s_i \in R^d, \quad i = 1,\ldots,n \end{aligned} \tag{14}$$

where the vector $s = [s_1\, s_2\, \ldots\, s_n]^T$ represents a design variable vector and F is the feasible set in the design space R^n which is defined as the set of design variables that satisfy the constraint functions $g(s)$ in the form:

$$F = \{s \in R^n \,|\, g_j(s) \le 0 \quad j = 1,\ldots,k\} \tag{15}$$

In most cases there is no unique point that would give an optimum for all m criteria simultaneously. Thus the common optimality condition used in single-objective optimization must be replaced by a new concept, the so called Pareto optimum: A design vector $s^* \in F$ is Pareto optimal for the problem of eq. (14) if and only if there is no other design vector $s \in F$ such that:

$$\begin{aligned} &f_i(s) \le f_i(s^*) \text{ for } i = 1,\ldots,m \\ &\text{with } f_i(s) < f_i(s^*) \text{ for at least one objective } i \end{aligned} \tag{16}$$

The solutions of optimization problems with multiple objectives constitute the set of the Pareto optimum solutions. The problem of eq. (14) can be considered as solved after the set of Pareto optimal solutions has been determined. In practical applications however, the designer seeks for a unique final solution. Thus a compromise should be made among the available Pareto optimal solutions.

5.1 Linear Weighting Method

The Linear Weighting Method combines all the objectives into a single scalar parameterized objective function by using weighting coefficients. If w_i, $i=1,2,\ldots,m$ are the weighting coefficients, the problem of eq. (12) can be written as follows:

$$\min_{s\in F} \sum_{i=1}^{m} w_i\, f_i(s) \tag{17}$$

with no loss of generality the following normalization of the weighting coefficients can be employed:

$$\sum_{i=1}^{m} w_i = 1 \tag{18}$$

By varying the weights it is possible to generate the set of Pareto optimum solutions for the problem of eq. (13). The values of the weighting coefficients are adjusted according to the importance of each criterion. Every combination of those weighting coefficients correspond to a single Pareto optimal solution, thus, by performing a set of optimization processes using different weighting coefficients it is possible to generate the full set of the Pareto optimal solutions.

5.2 Evolution Strategies for structural multi-objective optimization problems

The application of evolutionary algorithms in multi-objective optimization problems has attracted the interest of a number of researchers in the last ten years due to the difficulty of conventional optimization techniques, such as gradient based methods, to be extended in order to handle multi-objective optimization problems. ES, however, have been recognized to be more suitable for multi-objective optimization problems since the beginning of their development [17, 18]. Multiple individuals can search for multiple solutions simultaneously, taking advantage of any similarities available in the family of possible solutions to the problem.

In our implementation, where the weighting method is used, in order to generate a set of Pareto optimal solutions, the optimization procedure initiates with a set of parent design vectors needed by the ES optimizer and a set of weighting coefficients for the combination of all objectives into a single scalar parameterized objective function. These weighting coefficients are not set by the designer but are being systematically varied by the optimizer after a Pareto optimal solution has been achieved. There is an outer loop which systematically varies the parameters of the parameterized objective function, and is called decision making loop. The inner loop is the classical ES procedure, starting with an initial set of parent vectors. If any of these parent vectors gives an infeasible design then this parent vector is modified until it becomes feasible. Subsequently, the offsprings are generated and checked whether they are in the feasible region. According to the *(μ+λ)* selection scheme, in every generation the values of the objective function of the parent and the offspring vectors are compared and the worst vectors are rejected, while the remaining ones are considered to be the parent vectors of the new generation. On the other hand, according to the *(μ,λ)* selection scheme only the offspring vectors of each generation are used to produce the new generation. This procedure is repeated until the chosen termination criterion is satisfied. The number of parents and offsprings involved affects the computational efficiency of the multi-membered ES scheme discussed in this work. It has been observed that when the values of μ and λ are equal to the number of the design variables, better results are produced.

The ES algorithm combined with the standard methods can be stated as follows:

Outer loop - Decision making loop

Set the parameters w_i of the parameterized objective function

Inner loop - ES loop

1. *Selection step:* Selection of s_i (i = 1,2,...,μ) parent vectors of the design variables
2. *Analysis step*
3. *Evaluation of parameterized objective function*
4. *Constraints check:* All parent vectors become feasible
5. *Offspring generation:* Generate s_j, (j=1,2,...,λ) offspring vectors of the design variables
6. *Analysis step*
7. *Evaluation of the parameterized objective function*
8. *Constraints check:* If satisfied continue, else change s_j and go to *step 5*
9. *Selection step:* Selection of the next generation parents according to (μ+λ) or (μ,λ) selection schemes
10. *Convergence check:* If satisfied stop, else go to *step 5*

End of Inner loop

End of Outer loop

6. Test Examples

6.1 Six-storey space frame – RBDO test example

One 3-D building frame has been tested in order to illustrate the efficiency of the proposed methodologies for reliability-based sizing optimization problems. The cross section of each member of the two space frames considered is assumed to be a W-shape and one design variable is allocated for each member. The objective function is the weight of the structure. The deterministic constraints are imposed on the inter-storey drifts and, for each group of structural members, on the maximum non-dimensional ratio of eqs. (6) which combines axial forces and bending moments.

The probabilistic constraint is imposed on the probability of structural collapse due to successive formation of plastic nodes and is set to p_a=0.001. The probability of failure caused by uncertainties related to material properties, geometry and loads of the structures is estimated using MCS with the Importance Sampling technique. External loads, yield stresses, elastic moduli and the dimensions of the cross-sections of the structural members are considered as random variables. The loads follow a log-normal probability density function, while random variables associated with material properties and cross-section dimensions follow a normal probability density function. The required importance sampling function $g_x(x)$ for the loads is assumed to follow a normal distribution. In the tables showing the results of the test examples, *DBO* stands for the conventional Deterministic Optimization approach, *RBDO* stands for the conventional Reliability-Based Design Optimization approach, while *RBDO-NNi* corresponds to the proposed Reliability-Based Optimization with NN incorporating algorithm i (i=1,2).

Random variable	Probability density function	Mean value	Standard deviation
E	N	200	0.10E
σ_y	N	25.0	$0.10\sigma_y$
Design variables	N	s_i	$0.1s_i$
Loads	Log-N	6.4	0.20

Table 1: Characteristics of the random variables for the six-storey frame

This example consists of 63 elements with 180 degrees of freedom as shown in Figure 1. The length of the beams and the columns of the frame is L_1=7.32 m and L_2=3.66 m respectively. The structure is loaded with a gravity load of 19.16 kPa on all floor levels and a lateral load of 110 kN applied at each node in the front elevation along the z direction, acting as the basic load. The members of the structure are divided into five groups, as shown in Figure 4, each one having one design variable. The deterministic constraints are eleven, two for the stresses of each element group and one for the inter-storey drift. The type of probability density functions, mean values, and variances of the random parameters are presented in Table 1. The mean value for each geometric variable (i.e. the cross-sectional dimensions) is taken as the value the current design step of the corresponding variable s_i. The load-displacement curve of a node in the top-floor of the frame is depicted in Figure 2, corresponding to the design vector ($W_{12\times26}$, $W_{12\times33}$, $W_{12\times87}$, $W_{12\times87}$, $W_{10\times60}$) with a probability of failure equal to 8.68%.

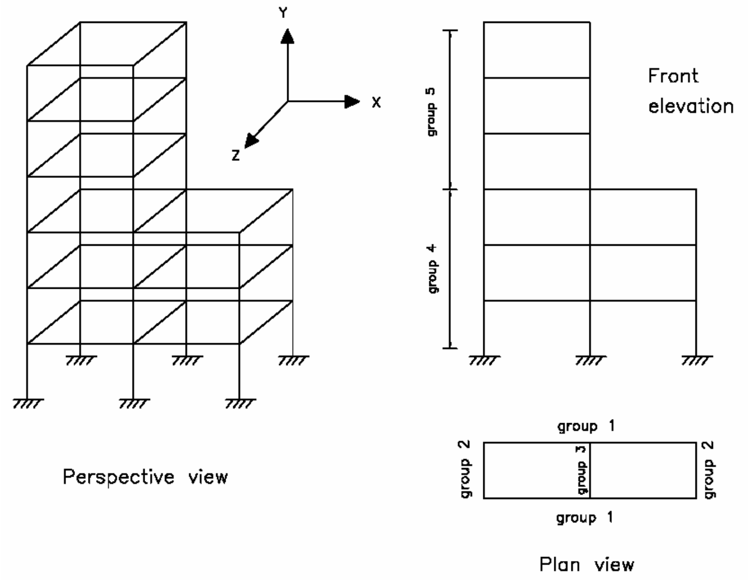


Figure 1: Description of the six-storey frame

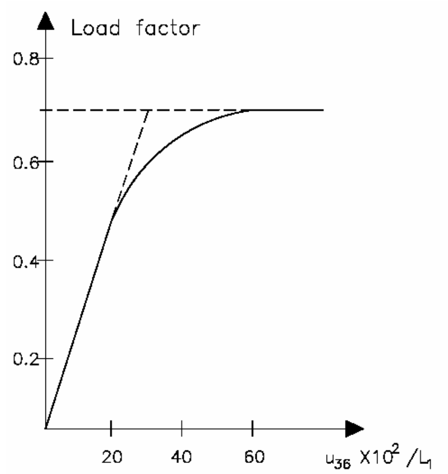


Figure 2: Load-displacement curve for the six-storey frame

For this test example the (μ+λ)-ES approach is used with μ=λ=5 following the rule that μ,λ should be equal to the number of the design variables. A sample size of 500, 1,000 and 5,000 simulations have been examined for the MCS with the importance sampling technique [6, 15], in order to study the influence of the number of simulations on the optimization process. As can be observed from Table 2 the probability of failure for the deterministic optimum is unacceptable since it exceeds substantially the accepted value 10^{-3}. On the other hand, the optimum weight achieved by the RBDO is 16% more than the deterministic one. For the application of the RBDO-NN1 methodology the number of NN input units is equal to the number of design variables, whereas one output unit is needed, according to both deterministic and probabilistic constraints. The output unit takes the values 1 or 0, corresponding to a feasible or infeasible design vector, respectively. Consequently the NN configuration implemented in this case has one hidden layer with 10 nodes resulting in a 5-10-1 NN architecture used for all runs. The training set consists of 100 training patterns chosen based on the requirement that the full range of the design space should be represented in the training procedure.

For the application of the RBDO-NN2 methodology the number of NN input units is equal to the number of the random variables, whereas one output unit is needed corresponding to the critical load factor. Consequently the NN configuration results in a 3-7-1 NN architecture which is used for all runs. The number of conventional step-by-step limit analysis calculations performed for the training of NN is taken 60 corresponding to different groups of random variables properly selected from the random field. As can be seen in Table 2 the proposed RBDO-NN2 optimization scheme manages to achieve the optimum weight in one third of the CPU time required by the conventional RBDO procedure. The number of MCS in the case of NN2 scheme can be extremely large without affecting its computational efficiency due to the trivial computing time required by the NN to perform one Monte Carlo simulation. The difference on the computational time needed by the NN1 methodology for different number of simulations, compared to NN2, is due to the fact that in the first methodology the computational time for the generation of the training set depends on the number of MC simulations.

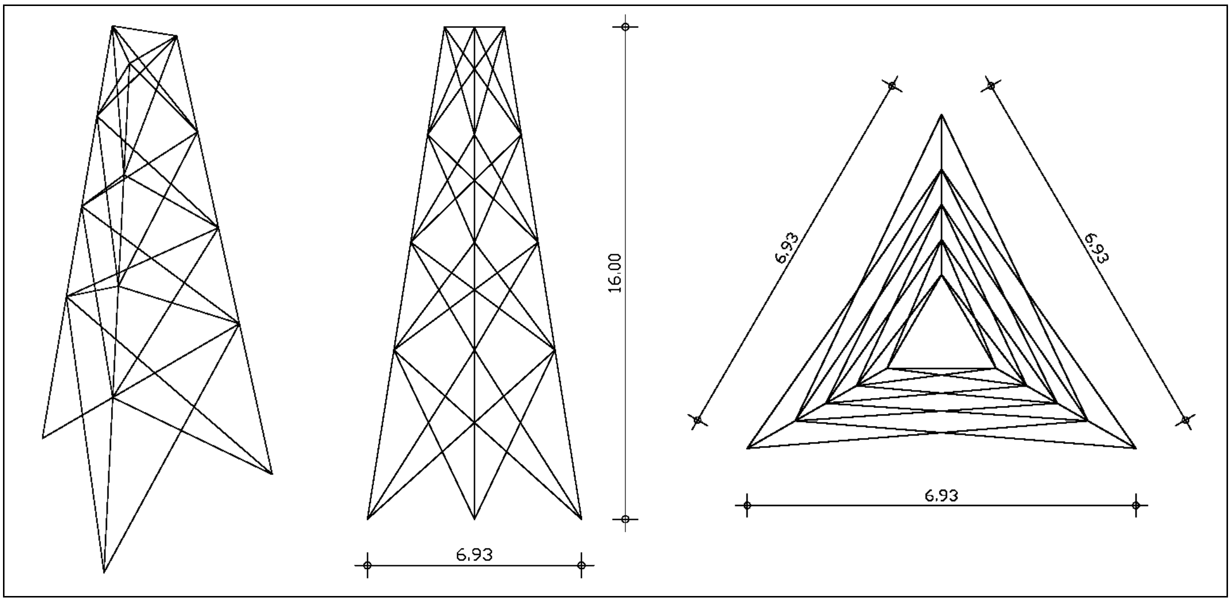
Optimization procedure	ES Generations	p_f**	Optimum weight (kN)	Time (h)
DBO	43	$0.171\ 10^{-0}$	727	0.05
RBDO (500 siml.)	65	$0.105\ 10^{-2}$	869	7.6
RBDO-NN1 (500 siml.)	64	$0.105\ 10^{-2}$	873	2.7
RBDO-NN2 (500 siml.)	65	$0.105\ 10^{-2}$	869	3.6
RBDO (1000 siml.)	68	$0.101\ 10^{-2}$	875	16.3
RBDO-NN1 (1000 siml.)	69	$0.101\ 10^{-2}$	875	5.3
RBDO-NN2*	66	$0.97\ 10^{-3}$	881	5.0
RBDO (5000 siml.)	68	$0.101\ 10^{-2}$	875	81.1
RBDO-NN1 (5000 siml.)	69	$0.101\ 10^{-2}$	875	26.5
RBDO-NN2*	66	$0.97\ 10^{-3}$	881	5.0

*For 100,000 simulations
**For 100,000 simulations using the NN2 scheme

Table 2: Performance of the methods for the six-storey frame

6.2 39-bar truss – RDO test example

A three dimensional 39-bar truss shown in Figure 3 is considered for presenting the efficiency of the proposed RDO methodology. The height of the structure is 16 m (Figure 3b), while its basis is an equilateral triangle of side 6.93 m (Figure 3c).



(a) (b) (c)

Figure 3: Three Dimensional 39-bar truss example (a) 3D view, (b) Side view, (c) Top view

Two objective functions are used, the weight and the variance of the response of the structure, under the constraints on stresses and displacements imposed by the design codes [10,11]. Due to engineering practice demands, the members are divided into groups having the same design variables. This linking of elements results in a trade-off between the use of more material and the need of symmetry and uniformity of structures due to practical considerations. Furthermore, it has to be taken into account that due to manufacturing limitations the design variables are not continuous but discrete since cross-sections belong to a certain pre-defined set provided by the manufacturers. Thus the design variables considered are the dimensions of the members of the structure, four groups in total, taken from the Circular Hollow Section (CHS) table of the Eurocode. For each design variable, two stochastic variables are assigned: The external diameter d and the thickness *t* of the circular hollow section. A vertical load V=2kN is applied to all nodes, while a probabilistic horizontal load *F* of mean value 8 kN is applied to the top nodes at the x-direction.

The types of probability density functions, the mean values, and the variances of the random parameters are shown in Table 3. For this test case the (μ+λ)-ES approach is used with μ=λ=5, while a sample size of 1,000 simulations is taken for the MCS.

		Probability Density Function	**Mean value μ**	**Standard Deviation σ**	**σ/μ**	**95% of values within**
E (kN/m^2)	Young's Modulus	Normal	2.10E+08	1.50E+07	7.14%	(1.81E+08, 2.39E+08)
σ_y (kN/m^2)	Allowable stress	Normal	355000	35500	10.00%	(2.85E+05, 4.25E+05)
F (kN)	Horizontal loading	Normal	8	3	37.50%	(2.12, 13.88)
d	CHS Diameter	Normal	d_i *	0.02 d_i	2%	(0.9608 d_i, 1.0392 d_i)
t	CHS Thickness	Normal	t_i *	0.02 t_i	2%	(0.9608 t_i, 1.0392 t_i)

* Taken from the Circular Hollow Section (CHS) table of the Eurocode, for every design

Table 3: Characteristics of the random variables

The resultant Pareto front curve is depicted in Figure 4, with the weight of the structure and the standard deviation of the horizontal displacement on the horizontal and vertical axis, respectively. The Pareto front curve shows a strong contradiction between the two objective functions in question.

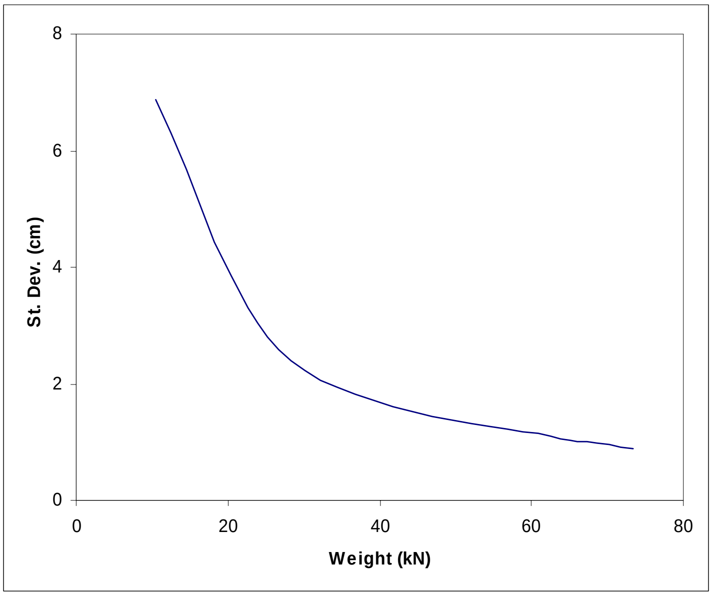


Figure 4: Pareto front curve

Conclusions

In most cases optimum design of structures is based on deterministic parameters and is focused on the satisfaction of the associated deterministic constraints. So far many articles have been devoted to this research field and efficient methods have been presented. When many random factors affect the design, the manufacturing and the life of a structure, the deterministic optimum cannot be considered as a realistic optimum design as a number of uncertain parameters have an important influence on the structural behavior. In order to find a more realistic optimum the designer has to take into account all necessary random parameters.

The aim of the proposed RBDO procedure was threefold; to reach an optimized design with controlled safety margins with regard to various model uncertainties, while at the same time minimize the weight of the structure and also reduce substantially the required computational effort. The solution of realistic RBDO problems in structural mechanics is an extremely computationally intensive task. In the test examples considered the conventional RBDO procedure was found over sixty times more expensive than the corresponding deterministic optimization procedure. The goal of decreasing the computational cost by at least one order of magnitude was achieved using: (i) NN predictions to perform both deterministic and probabilistic constraints check, or (ii) NN predictions to perform the structural analyses involved in MCS.

Evolution Strategies can be considered as an efficient tool for multi-objective design optimization of structural problems and in particular for the robust design sizing optimization problem. The proposed two stages evolution strategies method for treating multi-objective optimization problems proved to be a robust and reliable optimization tool. The deterministic based formulation of this structural optimization problem would converge to an optimum solution with the minimum weight, yet the resultant structural response would vary widely, and consequently the quality of the final design would be in doubt. In order to account for the randomness of parameters that affect the response of the structure, an RDO formulation of the optimization problem has to be used as shown in this work.

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