5th GRACM International Congress on Computational Mechanics
Limassol, 29 June – 1 July, 2005

ROBUST DESIGN OPTIMIZATION OF STEEL STRUCTURES

Vagelis Plevris, Nikos D. Lagaros, and Manolis Papadrakakis

Institute of Structural Analysis & Seismic Research
National Technical University of Athens
Zografou Campus, Athens 15780, Greece
e-mail: {vplevris, nlagaros, mpapadra @central.ntua.gr}

Keywords: Multi-objective, Optimization, Robust design, Monte Carlo, Latin Hypercube, Cascade evolutionary algorithms.

Abstract. *In real world engineering applications, the uncertainties are inherent and the scatter of structural parameters from their nominal ideal values is unavoidable. In deterministic based structural sizing optimization problems the aim is to minimize the weight or the cost of the structure, taking into account certain behavioral constraints on stresses and displacements, as imposed in a deterministic manner by the design codes. On the other hand, stochastic performance measures that involve various reliability requirements are being taken into consideration in many contemporary engineering applications. Reliability is defined as the probability of the system to meet the design demands during its life time. In structural optimization, stochastic performance measures can be taken into account using two distinguished formulations, Robust Design Optimization (RDO) and Reliability-Based Design Optimization (RBDO).*

In the case of a RDO problem, the uncertainties play a dominant role. Compared to the basic Deterministic Based Optimization (DBO) formulation, a RDO formulation yields a design with a state of robustness, so that its performance is the least sensitive to the variability of the uncertain parameters. In this work, the optimum design achieved based on a deterministic formulation is compared with the ones obtained employing a robust design formulation, with reference to the structural weight, the variance of the response and the probability of violation of the constraints.

1 INTRODUCTION

Although a great deal of studies has been proposed during the last three decades for structural optimization, those devoted to RDO are rather limited. In the present work, the non-dominant Cascade Evolutionary Algorithm (CEA)-based multi-objective optimization scheme is proposed for the solution of structural RDO problems, together with an improved handling of the multi-objective optimization problem, and is compared to the Linear Weighting Sum (LWS) method. The stochastic finite element problem is solved using the Monte Carlo simulation method combined with the Latin Hypercube Sampling (LHS) technique in order to reduce the number of simulations needed for the calculation of the required statistical quantities. Up to one hundred LHS simulations proved to be sufficient, for the test cases considered for calculating the statistical quantities. Furthermore, the advantages of the proposed cascade multi-objective optimization methodology over the classical LWS method and the importance of considering the variance of the structural response as a criterion are demonstrated. A real-scale truss structure has been examined subject to constraints imposed by the Eurocode 3 [7].

2 STOCHASTIC FINITE ELEMENT ANALYSIS

During the last two decades much progress has been achieved on stochastic finite element methods [18, 19]. However, comparatively few studies have been performed in structural optimization taking into account uncertain parameters. For the solution of stochastic finite element analysis problems, a number of methods have been proposed that can be classified into statistical and non-statistical ones. In this work, a statistical method and in particular Monte Carlo Simulation combined with the Latin Hypercube Sampling is employed.

2.1 Monte Carlo Simulation (MCS) Method

The MCS method is particularly applicable for the stochastic analysis of structures when an analytical solution is not attainable. This is mainly the case in problems of complex nature with a large number of uncertain variables, where all other stochastic analysis methods are inapplicable. Despite the fact that the mathematical formulation of the MCS is simple, the method has the capability of handling practically every possible case regardless of its complexity and the variation of the uncertain variables. The MCS method has proven to be efficient [16] for the calculation of the statistical quantities in the framework of a RBDO problem.

For the structural stochastic analysis problems examined in this study, the probability of violation of the behavioral constraints and the probability of failure are calculated along with the mean value and the variance of a characteristic nodal displacement that represents the response of the structure.

2.2 Latin Hypercube Sampling

The Latin Hypercube Sampling (LHS) method was introduced by MacKay et al. [13] in an effort to reduce the required computational cost of purely random sampling methodologies. Latin hypercube sampling is a strategy for generating random sample points ensuring that all portions of the random space in question are properly represented. LHS is generally recognized as one of the most efficient size reduction techniques. The basis of LHS is a full stratification of the sampled distribution with a random selection inside each stratum. In consequence, sample values are randomly shuffled among different variables. A Latin hypercube sample is constructed by dividing the range of each of the n_r uncertain variables into N non-overlapping segments of equal marginal probability. Thus, the whole parameter space, consisting of N parameters, is partitioned into N^{n_r} cells. A single value is selected randomly from each interval, producing N sample values for each input variable. The values are randomly matched to create N sets from the N^{n_r} space with respect to the density of each interval for the N simulation runs. The advantage of the LHS approach is that the random samples are generated from all the ranges of possible values.

3. MULTI-OBJECTIVE OPTIMIZATION

In many practical applications a single criterion rarely gives a representative measure of the actual structural performance, as several conflicting and usually incommensurable criteria have to be taken into account simultaneously. The optimization problem with more than one objective is called as multi-criteria, multi-objective or vector optimization problem [4].

3.1 Formulation of the multi-objective optimization problem

In general, the mathematical formulation of a multi-objective problem that includes a set of n design variables, a set of m objective functions and a set of k constraint functions can be defined as follows

$$
\begin{aligned}
&\min_{\mathbf{s}\in F} && [f_1(\mathbf{s}), f_2(\mathbf{s}), \ldots, f_m(\mathbf{s})]^T \\
&\text{subject to} && g_j(\mathbf{s}) \le 0 \quad j=1,...,k \\
& && s_i \in R^d, \quad i=1,...,n
\end{aligned}
\tag{1}
$$

where the vector $\boldsymbol{s} = [s_1\ s_2\ \ldots\ s_n]^T$ represents a design variable vector and F is the feasible region, a subspace of the design space R^n for which the constraint functions $g(\boldsymbol{s})$ are satisfied

3.2 Domination and non-domination

In single objective optimization problems the feasible set F can be ordered univocally according to the value of the objective function. For example, in the case of the minimization problem of $f(\mathbf{s})$, two solutions $\mathbf{s}_a$ and $\mathbf{s}_b \in$ F can be classified using the condition $f(\mathbf{s}_a) < f(\mathbf{s}_b)$. In a multi-objective optimization problem two solutions $\mathbf{s}_a$ and $\mathbf{s}_b \in$ F cannot be classified in a univocal manner. The concept of the Pareto dominance is used for assessing the two solutions, which for a minimization problem can be defined as follows

$$
\begin{aligned}
&\mathbf{s}_a \text{ dominates } \mathbf{s}_b \text{ if } f_i(\mathbf{s}_a) < f_i(\mathbf{s}_b)\ \forall i=1,\ldots,m \\
&\mathbf{s}_a \text{ weakly dominates } \mathbf{s}_b \text{ if } f_i(\mathbf{s}_a) \le f_i(\mathbf{s}_b)\ \forall i=1,\ldots,m \\
&\mathbf{s}_a \text{ is indifferent to } \mathbf{s}_b \text{ otherwise}
\end{aligned}
\tag{2}
$$

Using the definition of Eq. (2), the Pareto optimality can be stated as follows: A solution $\mathbf{s}^* \in$ F is Pareto optimal if it is not dominated by any other feasible design.

3.3 Solving the multi-objective optimization problem

Several methods have been proposed for treating structural multi-objective optimization problems [3, 9, 11]. According to Marler and Arora [12] these methods can be divided into: (i) methods with a priori articulation of preferences, (ii) methods with a posteriori articulation of preferences and (iii) methods with no articulation of preferences. The proposed Cascade Evolutionary Algorithm-based (CEA) multi-objective optimization scheme belongs to the first category. This algorithm is compared to the Linear Weighting Sum method (LWS), also belonging to the a priori articulation of preferences. The LWS method, due to its simplicity, is the most widely implemented method for solving such problems. In both methods employed, the problem in finding the Pareto front curve is reduced into a sequence of parameterized single-objective optimization *subproblems*, using scalarizing functions.

In general, by using scalarizing functions, locally Pareto optimal solutions are obtained. Global Pareto optimality can be guaranteed only when the objective functions and the feasible region are both convex or quasi-convex and convex, respectively. For non-convex cases, such as the majority of structural multi-objective optimization problems, a global single objective optimizer must be implemented. Evolutionary Algorithms (EA) are considered as global optimizers since they are not prone to being trapped in local optima and therefore can be considered as the most reliable methods in approaching the global optimum for non-convex constrained optimization problems. For this reason an evolutionary algorithm has been considered in this study for the solution of the sequence of the parameterized single objective optimization problems.

3.4 Non-dominant multi-objective search using the Tchebycheff metric

The proposed Cascade Evolutionary Algorithm (CEA)-based optimization scheme combines the CEA methodology with a non-dominance search and the Tchebycheff metric.

3.4.1 Augmented weighted Tchebycheff problem

The augmented weighted Tchebycheff method belongs to the methods with a priori articulation of the preferences for treating the multi-objective optimization problem and unlike the LWS method, can be applied effectively to convex as well as non-convex problems [15]. The weighted Tchebycheff metric can generate any optimal solution, to any type of optimization problem [20]. In order to overcome weakly Pareto optimal solutions, the Tchebycheff method formulates the distance minimization problem as a weighted Tchebycheff problem

$$\min_{\mathrm{s}\in \mathrm{F}} \max_{i=1,\ldots,m} \left[\mathrm{w_i} \frac{\left| f_i(\mathbf{s}) - z_i^* \right|}{f_i(\mathbf{s})} + \rho \sum_{i=1}^{m} \frac{\left| f_i(\mathbf{s}) - z_i^* \right|}{f_i(\mathbf{s})} \right] \tag{3}$$

where ρ is a sufficiently small positive scalar (in this work ρ = 0.1). The weight parameters w_i are random numbers, uniformly distributed between 0 and 1. The weight parameters have to make a sum of 1, if not, they are updated according to the following expression:

$$\mathrm{w_i} = \begin{cases} \mathrm{w_i} + \dfrac{1-\sum_{\mathrm{i}}^{\mathrm{m}} \mathrm{w_i}}{\sum_{\mathrm{i}}^{\mathrm{m}} \mathrm{w_i}} \mathrm{w_i}, & \text{if } \sum_{\mathrm{i}}^{\mathrm{m}} \mathrm{w_i} \neq 1 \\ \mathrm{w_i}, & \text{if } \sum_{\mathrm{i}}^{\mathrm{m}} \mathrm{w_i} = 1 \end{cases} \tag{4}$$

3.4.2 CEA-based multi-objective optimization scheme

It is generally accepted that there is still no unique optimization algorithm capable of handling all optimization problems efficiently. Cascade optimization attempts to alleviate this deficiency by applying a multi-stage procedure in which various optimizers are implemented successively. In the present work the idea of cascading is implemented in the EA-context (CEA) for solving multi-objective structural optimization problems. In particular, the CEA method is employed for the solution of the sequence of parameterized single objective optimization problems. The resulting cascade evolutionary procedure consists of a number of optimization stages (*csteps*), each of which employs the same EA optimizer. In order to diversify the search paths followed by the same optimization algorithm during the cascade stages, the initial conditions of the individual optimization runs are suitably controlled by using a different initial design at each stage (each stage initiates from the end point of the previous stage) and a different seed for the random number generator of the EA procedure [1].

A non-dominant search is performed in the context of the CEA and the Tchebycheff metric in the sense that all non-dominated solutions attained so far are kept in a set called temporary Pareto set. The multi-objective optimization problems are decomposed into *subproblems* which are solved with independent runs (*nruns* in total) of the CEA methodology. Each subproblem is independent from the others and therefore all subproblems can be dealt with simultaneously. Furthermore, in every global generation a non-dominant search is applied for updating the temporary Pareto set. The global generation is achieved when all local generations of the independent CEA runs are completed. According to this procedure in every global generation a local Pareto front is produced which approaches the global one.

The optimization algorithm proposed in this study is denoted as: Non-dominant $CEATm(\mu+\lambda)_{nruns,csteps}$ where μ, λ are the number of the parent and offspring vectors used in the ES optimization strategy, *nruns* is the number of independent CEA runs and *csteps* is the number of cascade stages employed. The proposed optimization scheme can be easily applied in two parallel computing levels, an external and an internal one. The multi-objective optimization problem is converted into a series of single objective optimization problems. The solution of each subproblem can be performed concurrently constituting the *external parallel computing level*. On the other hand, the utilization of the natural parallelization capabilities of the CEA methodology within each independent run defines the *internal parallel computing level*.

4. ROBUST DESIGN OPTIMIZATION

In the present study the robust design versus the deterministic-based design optimization of large-scale 3D truss structures is investigated. The random variables chosen are the cross-sectional dimensions of structural members, the modulus of elasticity E, the yield stress σ_y, as well as the applied loading.

4.1 Deterministic-based optimization

In a Deterministic-Based Optimization (DBO) problem the aim is to minimize the weight of the structure under certain deterministic behavioral constraints. In this study three types of constraints are imposed to the sizing optimization problem considered: (i) Stress, (ii) Compression force (for buckling) and (iii) Displacement constraints. The stress constraint can be written as follows

$$\sigma_{max} \le \sigma_a, \quad \sigma_a = \frac{\sigma_y}{1.10} \tag{5}$$

where σ_y is the yield stress, σ_{max} is the maximum axial stress in each element group for all loading cases and σ_a is the allowable axial stress, all taken according to the Eurocode 3 [7] for design of steel structures. For members under compression an additional constraint is used

$$\left|P_{c,max}\right| \le P_{cc}, \quad P_{cc} = \frac{P_e}{1.05}, \quad P_e = \frac{\pi^2 EI}{L_{eff}^2} \tag{6}$$

where $P_{c,max}$ is the maximum axial compression force for all loading cases, P_e is the critical Euler buckling force in compression, taken as the first buckling mode of a pin-connected member, and L_{eff} is the effective length. The effective length is taken equal to the actual length. Similarly, the displacement constraints can be written as

$$\left|d\right| \le d_a \tag{7}$$

where d_a is the limit value of the displacement at a certain node or at the maximum nodal displacement.

4.2 Formulation of the robust design optimization problem

In a robust design sizing optimization problem an additional objective function is considered which is related to the influence of the random nature of some structural parameters on the response of the structure. In the present study the aim is to minimize both the weight and the variance of the response of the structure due to the uncertainty of the random parameters. This problem is treated as a two-objective optimization problem using the weighted Tchebycheff metric. The mathematical formulation of the RDO problem implemented in this study is as follows

$$\begin{aligned} &\min && \Phi(\mathbf{s}) \\ &\text{subject to} && g_j(\mathbf{s}) \le 0 \quad j = 1,\dots,k \\ & && s_i \in R^d, \quad i = 1,\dots,n \end{aligned} \tag{8}$$

where $\Phi(\mathbf{s})$ is the multi-objective function, which is expressed as

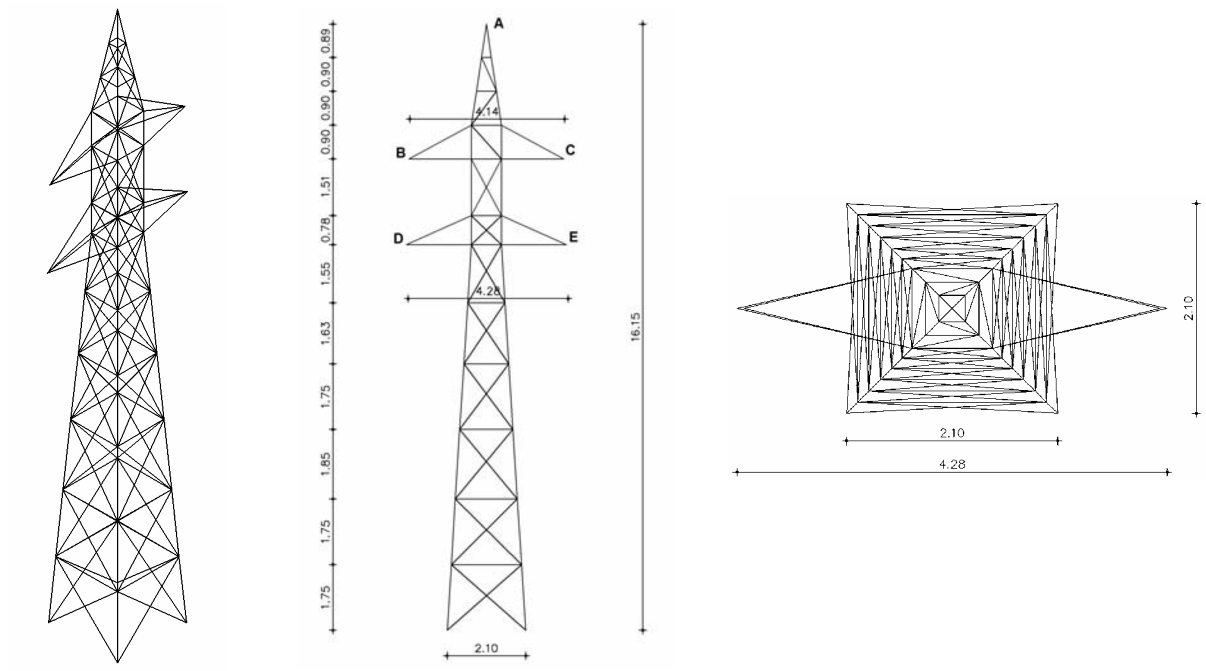
$$\Phi(\mathbf{s}) = \max\left[w_1 \frac{\left|f(\mathbf{s}) - z_1^*\right|}{f(\mathbf{s})} + \rho\sum_{i=1}^{m} \frac{\left|f(\mathbf{s}) - z_1^*\right|}{f(\mathbf{s})}, w_2 \frac{\left|\sigma_{u_i}(\mathbf{s}) - z_2^*\right|}{\sigma_{u_i}(\mathbf{s})} + \rho\sum_{i=1}^{m} \frac{\left|\sigma_{u_i}(\mathbf{s}) - z_2^*\right|}{\sigma_{u_i}(\mathbf{s})} \right] \tag{9}$$

where $f(\mathbf{s})$ is the weight of the structure and $\sigma_{u_i}(\mathbf{s})$ is the standard deviation of the response of the structure.

5. NUMERICAL TESTS

The numerical tests examined are performed in three stages. In the first stage the statistical methods used for the stochastic analysis are verified. The number of LHS simulations required for the calculation of the mean value and the standard deviation of the characteristic displacement that represents the structural response is compared with the corresponding number required by the basic MCS. In the second stage the advantages of the proposed non-dominant CEATm method over the LWS method are demonstrated through the comparison of the Pareto front curves obtained. In the third stage, the differences between DBO and RDO optimum designs, in terms of the final structural weight, the variance of response, the probability of violation of the constraints and the probability of failure, are illustrated.

The test example considered is a transmission tower, depicted in Figure 1. The design variables are the dimensions of the structural members, divided into seven groups, taken from the Equal Angle Section (EAS) table of the Eurocode. For each design variable, two stochastic variables are assigned: The length *L* and the width *t* of the legs of the section. The following loading vectors [Fx, Fy, Fz] in kN are applied to the structure: node A [-8.51, 0.00, -4.82], node B [-9.77, 0.00, -5.36], node C [-9.77, 0.00, -5.36], node D [-10.70, 0.00, -5.36] and node E [-10.70, 0.00, -5.36], while the type of probability density function, the mean value, and the variance of the random parameters are given in Table 1. A constraint maximum deflection of 200 mm is imposed.



(a) (b) (c)

Figure 1. Transmission tower: (a) 3D view, (b) Side view, (c) Top view

		PDF	Mean μ	St. Dev. σ	σ/μ	95% of values within
E (kN/m^2)	Young's Modulus	Normal	2.10E+08	1.50E+07	7.14%	(1.81E+08, 2.39E+08)
σ_y (kN/m^2)	Allowable stress	Normal	355000	35500	10.00%	(2.85E+05, 4.25E+05)
F (kN)	Nodal loading	Normal	μ_F	0.05 μ_F	5%	(0.902 μ_F, 1.098 μ_F)
L	Legs length	Normal	L_i *	0.02 L_i	2%	(0.961 L_i, 1.039 L_i)
t	Legs width	Normal	t_i *	0.02 t_i	2%	(0.961 t_i, 1.039 t_i)

* Taken from the Equal Angle Section (EAS) table of the Eurocode for every design

Table 1. Characteristics of the random variables

5.1 Efficiency of the stochastic analysis method

In the first stage of the numerical study, the performance of the LHS procedure in calculating the statistical parameters required during the RDO procedure compared to the basic MCS is examined, by measuring the influence of the number of simulations on the computed value of the variance of the characteristic displacement. The results, for randomly selected designs, shown in Figure 2 demonstrate the efficiency of the implemented LHS procedure. It can be observed from Figure 2 that 100 LHS compared to 500 MCS simulations are required in order to calculate the standard deviation of the structural response. It has to be stated that the number of simulations required may vary depending on the type of the structure, the loading conditions and the statistical characteristics of the structural parameters.

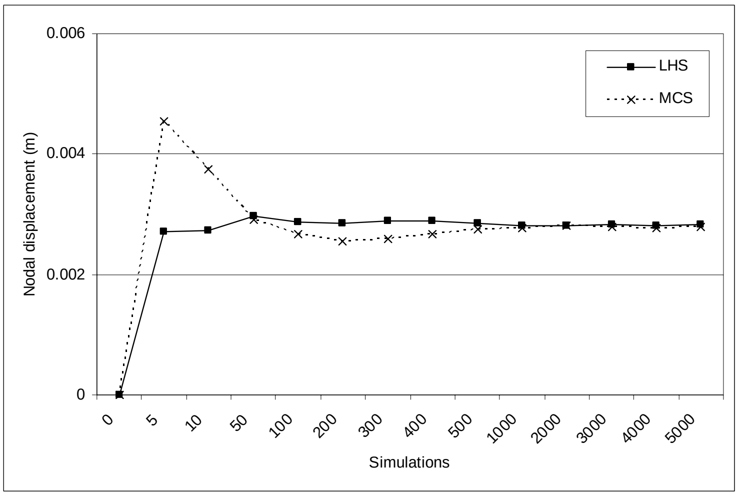
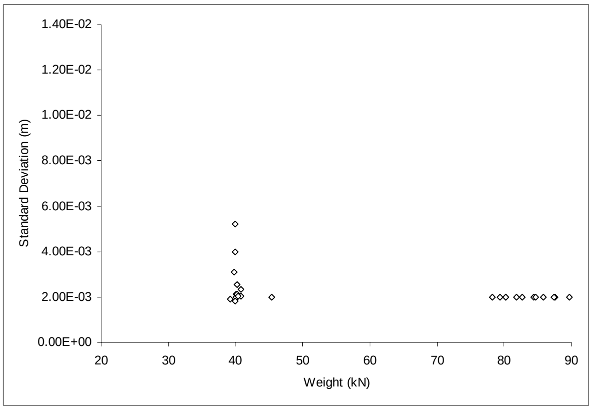


Figure 2. Efficiency of the LHS compared to the MCS in calculating the st. deviation of the structural response

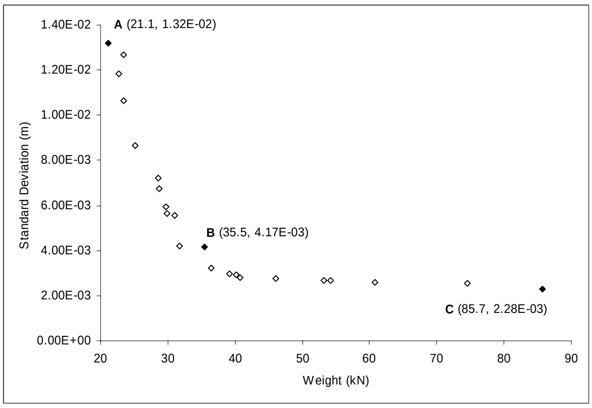
5.2 Comparison between LWS and CEATm

In the second stage of this study the advantages of the cascade evolutionary multi-objective optimization scheme using the Tchebycheff metric are demonstrated over the linear weighing sum method. The quality of the Pareto front curve can be assessed by the number of Pareto optimum solutions obtained and their distribution along the front curve. Well distributed solutions along the curve provide an indication of the efficiency of the multi-objective optimization method employed. The main drawback of the multi-objective optimization methods using scalarizing functions, such as the LWS, is the difficulty to fulfil these two requirements.

For the comparative study performed in this study the robust design optimization problem considered has been solved with the LWS method and the proposed non-dominant *CEATm* multi-objective optimization scheme. The LWS method has been implemented through two different runs with 10 and 30 points using the ES($\mu+\lambda$) optimization algorithm where $\mu = \lambda = 5$ are the number of parents and offsprings, respectively. For the non-dominant CEATm$(\mu+\lambda)_{nrun,csteps}$ optimization scheme the corresponding parameters are $\mu = \lambda = 5$, nrun = 10 and csteps = 3. The resultant Pareto front curves are depicted in Figure 3 (a) and (b) for the LWS and the CEATm, respectively. The horizontal axis corresponds to the structural weight while the vertical axis corresponds to the standard deviation of the characteristic node displacement.



(a)



(b)

Figure 3. The Pareto front curve obtained with (a) LWS - 30 points, (b) the non-dominant CEATm

The RDO multi-objective optimization problem is non-convex and the weakness of the LWS is obvious from the front curves of figure 3(a). Well distributed pairs of weighting coefficients do not correspond to equally well distributed Pareto optimum solutions along the front curve. On the other hand, the proposed CEATm optimization scheme manages to generate the Pareto front curve having a good distribution of the Pareto solutions along the front curve, seen in Figure 3(b).

5.3 Comparison between DBO and RDO solutions

In the third stage of this study the difference between DBO and RDO optimum designs is demonstrated in terms of the structural weight, the variance of the response and the probability of violation of the constraints. The resultant Pareto front curve, when the proposed optimization scheme is used, is shown in Figure 3(b). The two ends of the Pareto front curve represent two extreme designs. Point A corresponds to the deterministic-based optimum where the weight of the structure is the dominant criterion. Point C is the optimum when the standard deviation of the response is considered as the dominant criterion. The intermediate Pareto optimal solutions are compromise solutions between these two extreme optimum designs under conflicting criteria.

In Table 2 comparisons are performed for the three optimum designs A, B and C of Figure 3(b). The RDO(B) optimum design is achieved considering a compromise between the weight and the standard deviation. An important outcome of this investigation is that the DBO optimum design violates the constraints with probability equal to 1.1% and probability of failure equal to 0.6%. On the other hand, the probability of violation and the probability of failure, in the case of the compromise optimum design B, are computed one to two orders of magnitude lower compared to those corresponding to DBO designs. As a consequence of this reduced probability of violation and failure, an increase of 70% and 30% on the optimum weights achieved is observed in the case of RDO compared to the DBO. The value of the probability of violation is significantly lower in the case of optimum design C where the corresponding probability is 0.002%. However, the optimum weights achieved are 4 and 2 times more than the one obtained with the DBO formulations.

	DBO (A)	RDO (B)	RDO (C)
Weight (kN)	21.1	35.5	85.7
Standard Deviation (m)	$1.32\ 10^{-02}$	$4.17\ 10^{-03}$	$2.28\ 10^{-03}$
P_{viol} (%)	$1.1\ 10^{0}$	$7.0\ 10^{-2}$	$2.0\ 10^{-3}$
P_f (%)	$0.6\ 10^{0}$	$1.0\ 10^{-2}$	$0.8\ 10^{-3}$

Table 2. Characteristic optimal solutions

The hardware platform that was used in this work for the parallel computing implementation consists of a PC cluster with 25 nodes Pentium III in 500 MHz interconnected through Fast Ethernet, with every node in a separate 100Mbit/sec switch port. Message passing is performed with the programming platforms PVM working over FastEthernet. Two parallel processing schemes have been considered: *Parallel 1* corresponding to the exploitation of the parallel implementation of the optimization scheme and *Parallel 2* corresponding to the parallel implementation of the stochastic analysis involved in the optimization procedure. The computational performance for obtaining the multi-objective RDO Pareto front curve is compared in Table 3. The solution of the single-objective DBO(A) and RDO(C) problems, in sequential and parallel computing environments is examined. It can be seen that the computational time required for obtaining the RDO(C) optimum solutions is two orders of magnitude more than the corresponding time required to obtain the DBO(A) optimum solutions in sequential computing environment. This difference is reduced to one order of magnitude in parallel computing environment.

Formulation	Optim. Scheme	Generations	FE analyses	Time (s)		
				Sequential	Parallel 1*	Parallel 2*
DBO (A)	CEA(5+5)	103	627	63	19	-
DBO (C)	$CEATm(5+5)_{1,3}$	109	576	5214	349	229
RDO Par. F. C.	$CEATm(5+5)_{10,3}$	947	5528	51092	3127	2259

* In 25 processors

Table 3. Computational perfrormance

6. CONCLUSIONS

With the proposed multi-objective optimization scheme a uniform distribution of the Pareto optimum solutions along the front curve is achieved that is an indication of the efficiency of the optimization procedure. For the robust design optimization problem considered the proposed non-dominant *CEATm* multi-objective optimization methodology manages to generate the Pareto front curve with a good distribution of the Pareto solutions along the front curve.

The results obtained with the deterministic and the robust design optimization formulation underline the importance of minimizing the variance of the structural response when uncertain parameters are taken into account. For the test examples considered in this study, the probability of constraint violation of the DBO designs is computed two orders of magnitude greater than the corresponding probabilities for the RDO designs when the standard deviation of the response is considered as the dominant criterion to be minimized. On the other hand, the computational cost required for obtaining the DBO designs is two orders of magnitude in sequential and one order of magnitude in parallel computing environment less than the corresponding computing cost required for obtaining the RDO optimum solutions.

In the computational framework of robust design optimization of real-scale structures, the efficiency of the Latin hypercube sampling is also shown, requiring about a hundred of samples, in calculating the necessary statistical parameters. This part of the optimization procedure is very crucial since the computational cost of the RDO procedure is dependent directly on the number of simulations, while the reliability of the results obtained from the RDO procedure is influenced by the accuracy on the calculation of the statistical parameters.

REFERENCES

[1] D.C. Charmpis, N.D. Lagaros and M. Papadrakakis, Multi-database exploration of large design spaces in the framework of cascade evolutionary structural sizing optimization, Comput. Methods Appl. Mech. Engrg., In this Issue, 2005.

[2] W. Chen and K. Lewis, A robust design approach for achieving flexibility in multidisciplinary design, *AIAA Journal*, 37(8), 1999, 982-990.

[3] R.F. Coelho, H. Bersini and Ph. Bouillard, Parametrical mechanical design with constraints and preferences: application to a purge valve, Comput. Methods Appl. Mech. Engrg., 192(39-40), 2003, 4355-4378.

[4] C.A. Coello Coello, An updated survey of GA-based multi-objective optimization techniques, *ACM Computing Surveys*, 32(2), 2000, 109-143.

[5] I. Doltsinis and Z. Kang, Robust design of structures using optimization methods, Comput. Methods Appl. Mech. Engrg., 193, 2004, 2221-2237.

[6] Eurocode 1, Basis of design and actions on structures - Part 2-4: Actions on Structures - Wind actions, CEN, ENV 1991-2-4, 1995.

[7] Eurocode 3, Design of steel structures, Part1.1: General rules for buildings, CEN, ENV 1993-1-1, 1993.

[8] D.M. Frangopol and C.G. Soares (eds.), Reliability-oriented optimal structural design, Special Issue, Reliability Engineering & System Safety, 73(3), 2001, 195-306.

[9] D. Greiner, J.M. Emperador and G. Winter, Single and multiobjective frame optimization by evolutionary algorithms and the auto-adaptive rebirth operator, Comput. Methods Appl. Mech. Engrg., 193(33-35), 2004, 3711-3743.

[10] K.-H. Lee and G.-J. Park, Robust optimization considering tolerances of design variables, Computers and Structures, 79, 2001, 77-86.

[11] G.-C. Luh and C.-H. Chueh, Multi-modal topological optimization of structure using immune algorithm, Comput. Methods Appl. Mech. Engrg., 193(36-38), 2004, 4035-4055.

[12] R.T. Marler and J.S. Arora, Survey of multi-objective optimization methods for engineering, Structural and Multidisciplinary Optimization, 26(6), 2004, 369-395.

[13] M.D. McKay, R.J. Beckman and W.J. Conover, A comparison of three methods for selecting values of input variables in the analysis of output from a computer code, *Technometrics*, 21(2), 1979, 239-245.

[14] A. Messac and A. Ismail-Yahaya, Multi-objective robust design using physical programming, Struct. Multidisc Optim, 23, 2002, 357-371.

[15] K. Miettinen, Non linear multi-objective optimization, Kluwer Academic Publishers, 2002.

[16] M. Papadrakakis and N.D. Lagaros, Reliability-based structural optimization using neural networks and Monte Carlo simulation, Comput. Methods Appl. Mech. Engrg., 191(32), 2002, 3491-3507.

[17] M. Papadrakakis, V. Plevris, N.D. Lagaros and V. Papadopoulos, Robust design optimization of 3D truss structures using evolutionary computation, 6th WCCM in conjuction with APCOM'04, Beijing, China, Sept., 5-10, 2004.

[18] H. J. Pradlwarter, G. I. Schuëller and C. A. Schenk, A computational procedure to estimate the stochastic dynamic response of large non-linear FE-models, Comput. Methods Appl. Mech. Engrg., 192(7-8), 2003, 777-801.

[19] G.I. Schuëller (ed.), Computational stochastic structural mechanics and analysis as well as structural reliability, Special Issue, Comput. Methods Appl. Mech. Engrg., 2005.

[20] R.E. Steuer, Multiple criteria optimization: Theory computation and applications, John Wiley & Sons, 1986.